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Model for Estimating the Time of Concentration in Watersheds

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Abstract The time of concentration for a watershed is an essential parameter to the design of any hydrological project. In this study, a time of concentration estimation-model that uses variables obtained by monitoring rainfall-runoff events in a rural watershed of a tropical climate is proposed. In developing the model, the relationship between the time of concentration and independent variables was verified using a linear correlation matrix. Two variables with the highest correlation coefficient were selected to derive a model that can estimate the time of concentration. The Harmony Search (HS) optimization algorithm was employed to optimize the model parameters. The performance of the model was evaluated based on the Nash–Sutcliffe (NS) coefficient, which yielded a value greater than 0.80. The proposed model allows one to estimate the time of concentration using only the hydrograph of an event at the base level of a watershed, without the use of rainfall data.

Keywords Hydrographs · Surface runoff · Time factors · Hydrology · Parameters · Harmony search

1 Introduction

The time of concentration concept was first explored by Mulvany (1851) to describe the effects of rain on watersheds. The same author, in considering uniform rainfall over a drainage area, studied the time at which runoff reaches a maximum level, i.e., the time required for an entire

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watershed area to contribute to flow at the outlet point. The term was officially coined by Kuichling (1889), who studied rainfall-runoff relationships in populous neighborhoods of Rochester, New York (USA).

Several researchers (Simas and Hawkins 2002; Ogbonna 2004; and Li and Chibber 2008, among others) have derived eqs. to estimate the time of concentration. Each equation, derived empirically or analytically, emerged from studies conducted in a specific discipline and was modified to consider physical and hydrological characteristics of a given location. These eqs. have been frequently applied in the works of Kang et al. (2008); Upegui and Gutiérrez (2011), and Liang and Melching (2012), which included estimates of the time of concentration. Studies by Fang et al. (2008) and Sharifi and Hosseini (2011), among others, have identified variations in results obtained from various empirical equations. Fang et al. (2008) found wide differences between times of concentration estimated by different formulas that use watershed parameters. These average relative differences range from −38 to 207 %. According to Sharifi and Hosseini (2011), the time of concentration is an immeasurable variable, and designers determine it by use of formulas, which stem from varied surrogates and yield different results.

Definitions of the time of concentration have grown more sophisticated over time, according to a review of the perspectives considered. Eagleson (1970) defines it as the time required for surface runoff to reach a state of equilibrium. According to Chow et al. (1988), the time of concentration corresponds to the time required for an effective or excess raindrop to travel from the farthest point of a watershed to its outlet point. McCuen et al. (1984) refer to it is the time required for a drop of water to reach the farthest point of the watershed surface area (in the hydraulic path) before reaching the outlet point. The same author, who conducted studies using data for American urban watersheds, defines the time of concentration as the time elapsed from the end of effective rainfall to the end of the surface runoff. Su and Fang (2004) describe the time of concentration as the period running from the start of effective rainfall to the point at which flow reaches 98 % of equilibrium runoff. Wong (2005) defines it as the time elapsed since the start of effective rainfall to the time at which 95 % of the equilibrium runoff occurs. In this study the authors define the time of concentration as the time interval from the end of the rainfall period to the end of the surface runoff period.

Understanding watershed behaviors relating to the time of concentration allows for the prevention and mitigation of consequences of natural disaster and point source pollution in water resources. The changes in land use associated with urban development, replacing permeable soil by impervious surfaces, affect runoff. Gwenzi and Nyamadzawo (2014) cite that urbanization alone reduces the time of concentration, and increases peak discharge, and runoff volumes and velocity for the on-site and downstream hydrographs. According to Konrad (2003), with less storage capacity for water in urban basins and more rapid runoff, urban streams rise more quickly during storms and have higher peak discharge rates than do rural streams. Thus, the time of concentration in rural watersheds tends to be larger than time of concentration in urban watersheds.

According to Wong (2009), among all watershed response time parameters, the time of concentration is the most widely used parameter. Bondelid et al. (1982) showed that as much as 75 % of the total error in an estimate of the peak discharge can result from errors in the time of concentration. Recognizing the importance of the time parameters in hydrological design and water resource management, many studies have addressed the problem of estimating the time of concentration. Furthermore, the time of concentration reflects important physiographic features of a watershed. Pavlovic and Moglen (2008) argue that this parameter reflects the speed at which a drained area responds to rainfall events. Fang et al. (2008) note that if the time of concentration is underestimated, peak flow is overestimated and vice versa. The authors thus, stress the importance of accurate time of concentration estimations.

The majority of studies that have focused on the development of relevant equations to time of concentration have been based on data for watersheds of temperate regions. However, time of concentration and runoff generation processes are related to watershed responses to rainfall events. According to Dykes and Thornes (2000), as tropical soils exhibit varying characteristics, experiments that examine mechanisms of runoff generation in tropical and subtropical climates are necessary, as these mechanisms may differ under various conditions. This study thus formulates a methodology for estimating the time of concentration in rural watersheds of tropical climates using variables obtained through the monitoring of rainfall-runoff events. The sub-watershed used in this study is located in the capital city of Mato Grosso do Sul State, in the Brazilian Cerrado, and it is responsible for the most part of the urban water supply.

2 Materials and Methods

2.1 Study Area

The variables used in this study were obtained by monitoring rainfall-runoff events in the sub-watershed of the Guariroba stream, near the City of Campo Grande, Mato Grosso do Sul State, Brazil [parallels 20° 28′ and 20° 43′ (south latitude) and meridians 54° 29′ and 54° 11′ (west longitude)], covering an area of 360 km^2. This sub-watershed is tributary of Botas River, which is contributing Pardo River, Paraná River affluent. The local valley bottoms characterized by extensive occurrence of wet fields, veredas and other typical gallery forests of Cerrado.

The pedology of the drainage area is composed of 94.1 % orthic Quartzarenic Neosoil (RQo), 3.5 % hydromorphic Quartzarenic Neosoil (RQg) (both derived from sandstone rock) and 2.4 % dystrophic Red Latosol (DRL) of basalt origin. The study accounted for Hydrologic Response Unit 2 (HRU 2) within a 158 km^2 area with a base level located in the middle course of the sub-watershed (Latitude: 20°34′43,417″ S and Longitude: 54°19′58,335″ W), having an elevation between 480 m and 660 m. Rainfall time series were obtained from data recorded at two rainfall, located at similar elevation, stations that operate at sites 1 and 4 (Fig. 1). Furthermore, we used double mass analysis as consistency tool of rainfall data and it was seen the homogeneity of the stations 1 and 4.

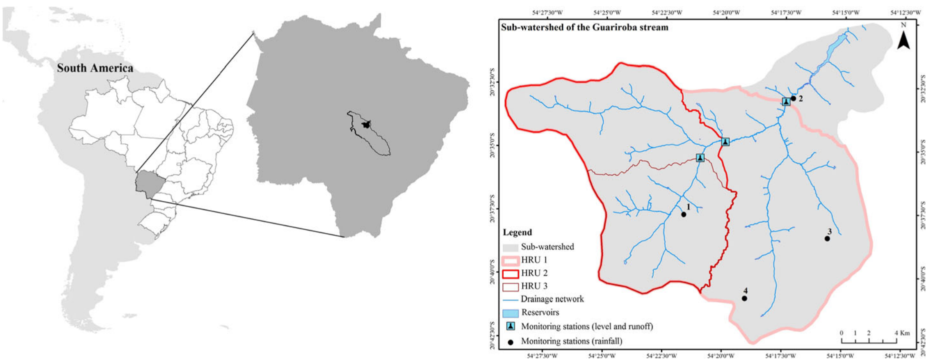

Fig. 1 Sub-watershed of the Guariroba stream, its HRUs and the locations of measurement stations

The climate of the region, according to the Köppen climate classification model, is of the Aw type, defined as a tropical, hot and humid climate. This climate is characterized by temperatures that exceed 18 ° C during the winter season (June to August). It features well-defined rainy periods (November to April) falling within the range of 750–2.000 mm and pronounced drought in the months of July to September. The annual average rainfall for this region is 1.500 mm.

2.2 Hydrological Data

Watercourse level and rainfall depth data were continuously recorded, at an interval of 5 min, with the use of a datalogger throughout the years of 2011 to 2014. The discharge measurements were made, at varying intervals depending on the timing of rainfall periods observations, at the HRU 2 outlet. Hydrological data were post-processed in a spreadsheet to study the events.

2.3 Determination of Variables

The runoff data were prepared by applying the rating curve to the water-level data measured continuously. A continuous series of rainfall and runoff events has been analyzed, out of which of 17 distinct rainfall events representing the most possible variations of intensities and durations has been selected. The rainfall-runoff events selected in this study were all independent ones, unaffected by any other previous events.

The time of concentration (Tc) and other variables required for the study (below defined) were estimated via a graphical analysis of hyetographs and hydrographs for the selected events. No universally accepted graphical method has yet been developed for measuring the final point of effective rainfall. Thus, the Tc was defined as the time interval from the end of the rainfall period to the end of the surface runoff period.

In this study, reference to runoff means storm water runoff or the runoff occurring during and immediately following a rainfall event. According to Haan et al. (1994), the volume of storm water runoff is equal to the volume of rainfall excess or effective precipitation. Thus, the area below the runoff curve, which is bounded by the line that connects the start and end points of surface runoff, corresponds to the runoff volume. The start point is indicated by the beginning of the rise in the water-level. The end point is initially determined by the probable range of the end of the direct surface runoff. This range is evidenced by a tendency of a decreasing line segment of the graph in semi-logarithmic scale. Dividing the runoff volume by the study area generates the height of effective rainfall (h_{ef}). Using the total height of accumulated rainfall for each event (h_t) and the height of effective rainfall, the percentage of abstractions (P_{abs}) can be obtained, as shown in Eq. 1.

$$P_{abs} = \frac{\left(h_t - h_{ef}\right)}{h_t}.100 \qquad (1)$$

Other variables obtained from the graphical analysis of the events were defined as follows: the peak runoff (Q_p) is the maximum runoff level recorded for each event; rain duration (T_d) is the total duration of each rainfall period that gave rise to each event; average rainfall intensity (i_{avg}) is the total accumulated rainfall level divided by the total duration of the rainfall period; maximum intensity (i_{max}) is the maximum intensity level recorded in mm/5 min; (T_{CG}) is the elapsed time from the start to the rainfall centroid; (T_r) denotes the time interval from the start of the maximum runoff period to the end of the surface runoff period; (T_b) denotes the time elapsed between the start and end of the surface runoff period; (T_p) is the time interval between the rainfall centroid and the start of the maximum runoff period; (T_a) denotes the time interval from the start of the surface runoff period to the start of the maximum runoff period.

When considering Tc as the dependent variable and the others as independent variables, an equation that relates all of these variables can be constructed. Therefore the Tc can be defined as function of $h_{t.}$, h_{ef}, P_{abs}, Q_p, T_d, i_{avg}, i_{max}, T_{CG}, T_r, T_b, T_p and T_a. Thus, the independent variables influencing on the dependent variable can be determined using Pearson correlation coefficient, as a measure of dependencies between variables (Kohnová and Szolgay 2003; Latt et al. 2015).

2.4 Correlation Analysis

The Pearson linear correlation coefficient (r), shown in Eq. 2, describes the linear correlation between two random variables (x, y) and is not dependent on the measurement unit.

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right] \cdot \left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}} \quad (2)$$

As this coefficient approaches 1 or −1, correlations in the observed data grow stronger. A correlation of zero denotes the absence of a linear relationship between variables (Dashtaki et al. 2009). Table 1 shows other degrees of correlation between variables.

Through correlation analysis, it is possible to evaluate the relationship between the independent variables and the dependent variable Tc. The relationship between the Tc and the independent variables was verified using a linear correlation matrix based on a 95 % confidence interval. The null hypothesis (Ho) was taken as the assertion that the dependent variable Tc can be influenced by the independent variables. Based on the matrix, the two variables with the highest correlation coefficients were used to define the model (Eq. 3).

$$T_c = K.\ V_1^m.\ V_2^n \quad (3)$$

Here, V_1 and V_2 are the selected variables, and K, m, and n are the equation parameters to be optimized.

2.5 Harmony Search

As originally proposed by Geem et al. (2001), the harmony search (HS) algorithm was conceptualized from the musical process of searching for a 'perfect state' of harmony, such as jazz improvisation. The same authors claim that jazz improvisation seeks a best state determined by aesthetic estimation, just as the optimization algorithm seeks a best state determined by evaluating

Table 1 Degrees of correlation between variables according to the Pearson coefficient

Pearson correlation coefficient	Degree of correlation
$0.90 < r \leq 0.99$	Very strong
$0.70 < r \leq 0.90$	Strong
$0.30 < r \leq 0.70$	Moderate
$0 < r \leq 0.30$	Weak

the objective function. According to Kougias and Theodossiou (2013) it is a music inspired method, imitating the music creation process in order to find optimal solutions in complicated problems.

The HS algorithm has been applied in various optimization problems, including those involving structural design (Saka 2007), economic uses of heat and power (Vasebi et al. 2007), water network projects (Geem 2009), groundwater management schemes (Ayvaz 2009), geotechnical stability problems (Cheng et al. 2008), and hydrological model parameter estimations (Geem 2010).

The fit between the estimated and observed times was verified using the objective function F_{obj}, which seeks to minimize the sum of squared differences between observed and estimated values (Eq. 4).

$$F_{obj} = \sum_{i=1}^{n} \left(T_{C_{obs}} - T_{C_e}\right)^2 \tag{4}$$

Here, n is the number of events, Tc_{obs} is the time of concentration obtained from a graphical analysis of hyetographs and hydrographs, and Tc_e is the time of concentration estimated via the proposed model. The fit between the observed and estimated values was assumed when the objective function was held at a minimum value, i.e., when the sequence of iterations did not improve the quality of results obtained, thereby ceasing the optimization process.

The quality of fit between observed and estimated time values was calculated using the Nash-Sutcliffe efficiency coefficient (Nash and Sutcliffe 1970; Patel and Ramachandran 2015; Liu et al. 2016), as shown in Eq. 5.

$$NS = 1 - \frac{\sum_{i=1}^{n} \left(T_{C_{obs}} - T_{C_e}\right)^2}{\sum_{i=1}^{n} \left(T_{C_{obs}} - \overline{T_{C_{obs}}}\right)^2} \tag{5}$$

Here, NS is the Nash-Sutcliffe efficiency coefficient; Tc_{obs} is the time of concentration obtained from a graphical analysis of hyetographs and hydrographs; Tc_e is the time of concentration estimated using the model; and $\overline{T_{Cobs}}$ is the average of all observed times of concentration.

The NS can range from negative infinity to 1.0. A negative sign denotes that the model exhibits a poor fit, suggesting that the difference between what was observed and estimated is even greater than the variance between observed values. When the value of NS reaches unity, this denotes a perfect fit between what was estimated and observed. Thus, a value that approaches unity denotes a more efficient and accurate model. Santhi et al. (2001) presented the following classification for this coefficient: when NS > 0.65, the model is deemed very good; when 0.54 < NS <0.65, the model is deemed good; and when 0.5 < NS < 0.54, the model is deemed satisfactory. According to Gottschalk and Motovilov (2000), the performance of a model is considered suitable if the NS value exceeds 0.75 and is considered acceptable if this value ranges between 0.36 and 0.75.

Figure 2 presents a flowchart of the proposed model.

3 Results and Discussion

Figure 3 presents the hydrograph of one of the 17 selected rainfalls events with the Tc and the end of the surface runoff period marked.

Independent variables determined from the graphical analysis of the hyetographs and hydrographs for the selected events are shown in Table 2.

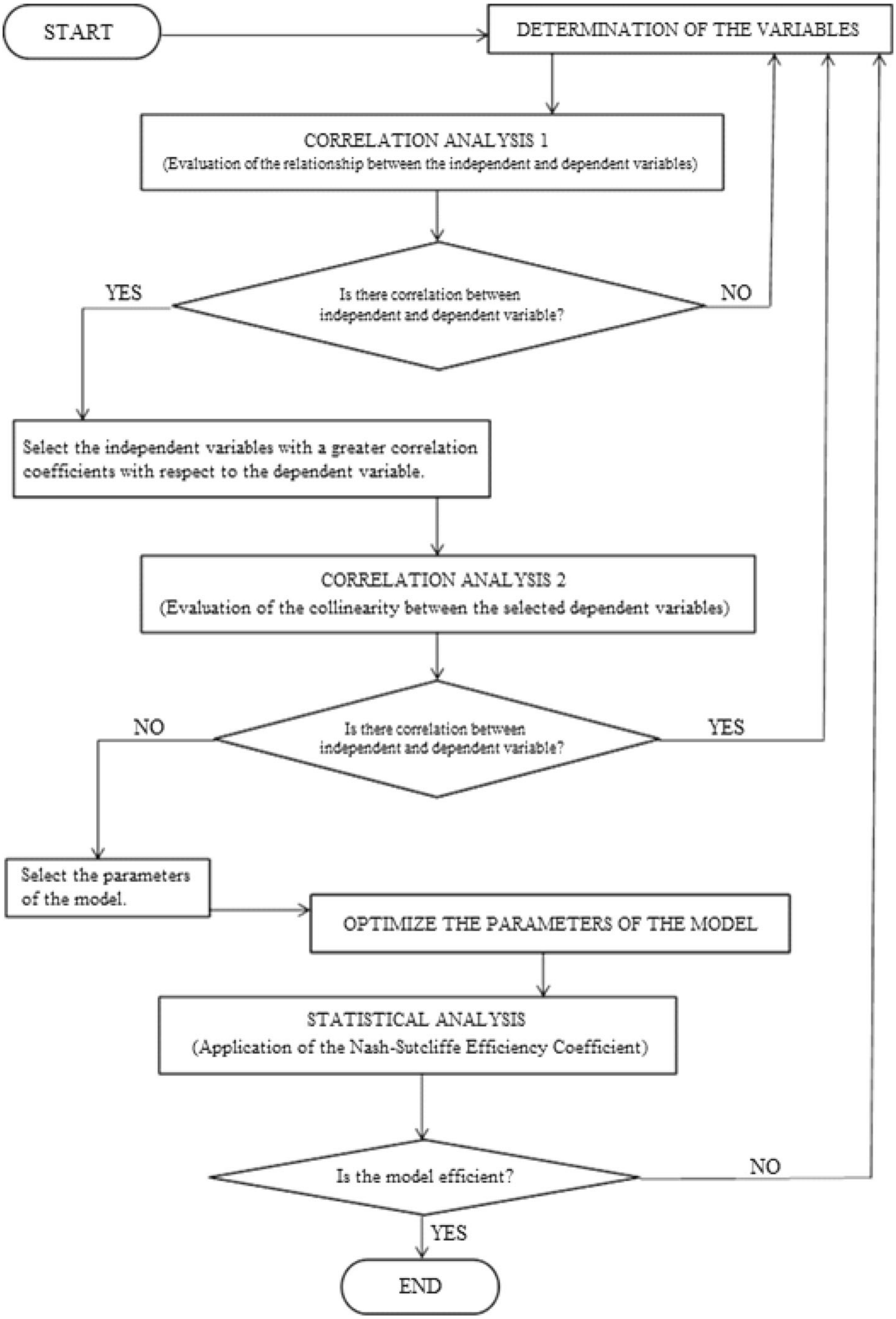

Fig. 2 Flowchart of the proposed model

Istok and Boersma (1986) studied the effect of previous rainfall events on surface runoff generation and found that relative to rainfall intensity, antecedent moisture has more significant

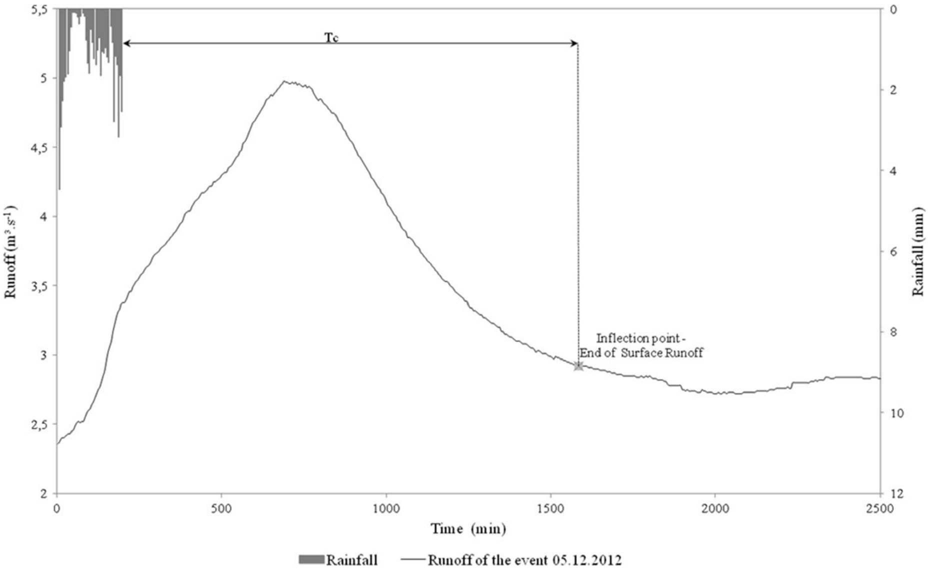

Fig. 3 Hydrograph of the 05/12/2012 event with the Tc and the end of the surface runoff period marked

effects on the volume of water drained. This result necessitated an analysis of P_{abs} and h_{ef}. The lowest value of P_{abs} (98.56 %) was observed on 11/11/13 (after no rainfall for the previous eight days) and the highest value (99.71 %) was observed on 2/28/13 (after no rainfall for the previous three days), and both were recorded during rainy seasons. The lowest value of h_{ef} (0.07 mm) was observed on 12/2/12 and the highest value of h_{ef} (0.68 mm) was observed on 5/12/12. It should be noted that these two events occurred following similar periods of zero precipitation (one day). Such observations show that in the study area, variations in time elapsed since preceding periods of rainfall do not influence surface runoff generation patterns. In the sub-watershed of the Guariroba stream, 0.74 % of total rains turn into direct surface runoff on average. It is understood that the high average value of P_{abs} (99.26 %) and low average value of h_{ef} (0.31 mm) can be attributed to the dominant class of soil present in the study area, as it Orthic Quartzarenic Neosoil (RQo) is derived from sandstone rock and exhibits high infiltration capacities and low retention capacities for moisture.

To a certain extent, were found variations in Tc values (18,20 h – 26,39 h) obtained from the events analysis. As the rainfall-runoff process in a basin is intrinsically nonlinear, it is practically impossible to determine the unique value of time of concentration to all rainfall-runoff selected events. Thus, assigning a unique value for the Tc of a particular watershed generates an estimate of the average Tc value for the watershed. Thus, methodologies for estimating Tc generate this average value.

The analysis of the simple linear correlation between the independent variables and the Tc resulted in the development of a linear correlation matrix (Table 3). In this table, the top row includes coefficient values, which represent the degree and direction of correlations. Values shown in parentheses below each coefficient help identify whether a correlation significantly differs from zero. The smaller this value is, the greater the chance of a linear correlation, i.e., the higher the certainty that the null hypothesis is not rejected.

According to the results obtained by McCuen et al. (1984), the intensity of effective rainfall is highly correlated with Tc. However, in this study, no significant correlation between

Table 2 Variables determined from the graphical analysis of the hyetographs and hydrographs

Item	Event	T_c (hours)	h_t (mm)	h_{ef} (mm)	P_{abs} (%)	Q_p ($m^3.s^{-1}$)	T_d (hours)	i_{avg} ($mm.h^{-1}$)	i_{max} (mm/5 min)	T_{CG} (hours)	T_r (hours)	T_b (hours)	T_p (hours)	T_a (hours)
1	11/14/2011	26.18	63.25	0.41	99.35	3.46	8.49	7.45	8.01	5.08	14.65	34.02	14.94	19.37
2	02/01/2012	20.42	37.25	0.13	99.66	3.03	3.11	11.96	8.48	0.45	6.84	23.04	16.25	16.20
3	02/03/2012	24.19	31.50	0.30	99.05	3.75	5.51	5.71	6.49	4.30	14.70	29.12	10.70	14.42
4	03/13/2012	23.21	17.75	0.19	98.91	3.25	6.28	2.82	2.04	1.43	14.20	27.24	13.86	13.04
5	05/12/2012	22.32	49.50	0.68	98.63	4.98	4.01	12.35	4.47	1.70	14.83	26.24	9.80	11.42
6	09/21/2012	23.54	63.75	0.42	99.34	3.85	7.47	8.54	5.70	2.46	14.93	30.51	13.63	15.58
7	12/02/2012	18.20	14.50	0.07	99.51	2.74	2.63	5.51	4.69	1.98	8.42	20.75	10.44	12.33
8	12/28/2012	24.17	67.00	0.52	99.22	4.24	8.08	8.29	8.81	4.83	19.84	30.08	7.59	10.24
9	12/30/2012	21.13	27.50	0.16	99.40	3.26	3.86	7.12	5.54	1.13	13.42	24.75	10.45	11.33
10	02/28/2013	23.54	76.75	0.22	99.71	3.19	7.17	10.70	7.31	1.48	12.75	30.12	16.48	17.38
11	03/24/2013	23.50	68.50	0.39	99.43	3.85	2.67	25.67	8.36	0.71	14.17	26.00	11.29	11.83
12	04/07/2013	22.64	37.75	0.33	99.13	4.18	3.62	10.44	4.26	0.92	13.58	25.84	11.74	12.25
13	06/02/2013	24.00	47.25	0.22	99.54	3.42	5.17	9.14	5.26	2.12	10.92	28.67	16.13	17.75
14	07/01/2013	23.57	20.50	0.16	99.21	3.00	8.99	2.28	3.00	2.59	15.42	32.39	14.55	16.97
15	10/04/2013	23.41	40.50	0.31	99.24	3.32	9.18	4.41	4.33	3.79	12.42	32.09	16.38	19.67
16	10/22/2013	23.60	35.00	0.15	99.58	2.78	4.24	8.26	3.61	1.83	19.01	26.84	7.00	7.83
17	11/11/2013	26.39	37.75	0.54	98.56	4.01	5.55	6.80	8.25	2.81	15.30	31.86	13.84	16.57
	Sum	394.00	736.00	5.20	1687.48	60.31	96.04	147.46	98.61	39.60	235.38	479.56	215.08	244.18
	Average	23.18	43.29	0.31	99.26	3.55	5.65	8.67	5.80	2.33	13.85	28.21	12.65	14.36
	Standart devitation	1.94	18.96	0.17	0.33	0.59	2.24	5.25	2.11	1.42	3.18	3.58	3.04	3.40

Table 3 Simple linear correlation matrix (r-Pearson)

	T_c (hours)	h_t (mm)	h_{ef} (mm)	P_{abs} (%)	Q_p ($m^3.s^{-1}$)	T_d (hours)	$i_{méd}$ ($mm.h^{-1}$)	i_{max} (mm/5 min)	T_{CG} (hours)	T_r (hours)	T_b (hours)	T_p (hours)	T_a (hours)
T_c (hours)	1												
h_t (mm)	0.442 (0.075)	1											
h_{ef} (mm)	0.526 (0.030)	0.528 (0.029)	1										
P_{abs} (%)	−0.365 (0.155)	0.216 (0.404)	−0.670 (0.003)	1									
Q_p ($m^3.s^{-1}$)	0.325 (0.203)	0.417 (0.096)	0.915 (0.000)	−0.664 (0.004)	1								
T_d (hours)	0.575 (0.016)	0.248 (0.338)	0.186 (0.476)	−0.102 (0.698)	−0.028 (0.916)	1							
$i_{méd}$ ($mm.h^{-1}$)	−0.028 (0.914)	0.582 (0.014)	0.280 (0.276)	0.211 (0.416)	0.342 (0.179)	−0.516 (0.034)	1						
i_{max}(mm/5 min)	0.264 (0.306)	0.637 (0.006)	0.362 (0.154)	0.154 (0.554)	0.242 (0.349)	−0.047 (0.859)	0.498 (0.042)	1					
T_{CG} (hours)	0.550 (0.022)	0.192 (0.461)	0.362 (0.153)	−0.221 (0.393)	0.148 (0.570)	0.699 (0.002)	−0.428 (0.087)	0.232 (0.370)	1				
T_r (hours)	0.613 (0.009)	0.260 (0.313)	0.477 (0.053)	−0.349 (0.170)	0.363 (0.152)	0.389 (0.123)	−0.064 (0.807)	−0.036 (0.892)	0.438 (0.079)	1			
T_b (hours)	0.868 (0.000)	0.385 (0.127)	0.403 (0.109)	−0.254 (0.325)	0.163 (0.532)	0.880 (0.000)	−0.292 (0.256)	0.130 (0.620)	0.688 (0.002)	0.499 (0.041)	1		
T_p (hours)	0.165 (0.527)	0.104 (0.692)	−0.194 (0.456)	0.162 (0.533)	−0.261 (0.311)	0.372 (0.141)	−0.132 (0.613)	0.063 (0.809)	−0.057 (0.829)	−0.570 (0.017)	0.360 (0.155)	1	
T_a (hours)	−0.102 (0.696)	0.272 (0.291)	−0.039 (0.883)	0.299 (0.243)	−0.105 (0.689)	0.016 (0.953)	0.107 (0.682)	0.362 (0.153)	0.393 (0.118)	−0.024 (0.926)	−0.075 (0.775)	−0.075 (0.775)	1

Table 4 Initial uncertainty range of the variables to be optimized

Variable	L_{low}	L_{up}
K	2.00	4.00
m	0.40	0.60
n	0.05	0.25

characteristic parameters of event rainfall and Tc was found. This may be attributable to the fact that interception and infiltration processes in rural watersheds are largely shaped by sandy soil effects on water balance levels.

Given that the correlations with a significance level of 5 % are acceptable, five significant correlations for Tc were deemed distinct. A strong correlation was found between Tc and T_b (0.868), and moderate correlations were found between Tc and h_{ef} (0.526), T_d (0.515), T_{CG} (0.550), and T_r (0.613). The sign of the coefficient denotes the proportionality of the relationship. If both variables tend to increase or decrease simultaneously, the coefficient is positive. If a variable tends to increase as the other decreases, the coefficient is negative.

The Pearson correlation coefficient of the relationship between Tc and T_b is 0.868, and the *p*-value is close to zero, indicating that the observed effect is not attributable to chance. Thus, it shows that Tc tends to increase as T_b increases. A similar relationship is found for Tc and T_r.

In cases in which independent variables are strongly correlated, a problem of multicollinearity arises. Collinear variables affect the interpretation of model coefficients. To prevent cases of multicollinearity, Naghettini and Pinto (2007) recommend eliminating one independent variable for each set of two independent variables with a correlation coefficient greater than 0.85. The strongest correlations were found between Tc and T_b and between Tc and T_r. Thus, collinearity probabilities for T_b and T_r were examined. At a 4.1 % significance level, i.e., within a 95 % confidence interval, it was verified that the correlation coefficient between T_b and T_r is 0.499, proving the non-collinearity of the analyzed variables.

Thus, it is proposed that Tc is a function of T_b and T_r, i.e., Tc = f (T_b, T_r); based on this assumption, the model is formulated (Eq. 6).

$$T_c = K.\ T_b^m.\ T_r^n \tag{6}$$

Table 5 Final values generated for the variable based on a 10-value harmonic memory

K	m	n	F_{obj}
3.651	0.494	0.076	11.835
3.652	0.494	0.076	11.835
3.652	0.494	0.076	11.839
3.657	0.494	0.076	11.842
3.652	0.494	0.076	11.843
3.652	0.494	0.077	11.845
3.658	0.494	0.076	11.848
3.654	0.494	0.077	11.862
3.658	0.494	0.076	11.866
3.655	0.493	0.076	11.886
3.657	0.493	0.076	11.906

The model parameters were optimized using the HS algorithm. Initially, an uncertainty range with lower and upper limits for values of K, m, and n was stipulated (Table 4).

A routine that reads the limits established for each variable and that generates a random value for these limits was written based on a harmonic memory parameter, NMH = 10, for each of the three variables involved in the optimization process, where the number of iterations, NIT = 100. After each iteration, the 10 best ordered sets were selected, and the lower and upper limits of each variable were used in the next iteration. This method was repeated until the objective function reached a minimum value, i.e., when the sequence of iterations did not further improve the quality of results obtained (Table 5).

In turn, the model for estimating Tc was formulated (Eq. 7).

$$T_c = 3,651\ T_b^{0,494}\, T_r^{0,076} \tag{7}$$

To determine the model's predictive capacity, the NS efficiency coefficient was calculated, and was found to be 0.804. These statistical analyses show that according to Gottschalk and Motovilov (2000), the performance of the model is appropriate and good, and according to Santhi et al. (2001) parameters, the model is deemed very good, achieving a Nash-Sutcliffe coefficient of 0.804.

4 Conclusion

Variations in periods elapsed since a previous rainfall period do not influence surface runoff generation patterns in watersheds similar the study area. The base time of the hydrograph and the time interval between the start of maximum runoff and the end of surface runoff constitute the main hydrological variables that affect the time of concentration in watersheds with features that reflect those of the study area. The proposed model allows one to estimate the time of concentration using only the hydrograph of an event at the base level of a watershed and without the use of any rainfall data. These results are promising and show that the applied methodology is efficient and effective in estimating time of concentration values for watersheds.

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Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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